The project is led by Dr. Markus Faulhuber and deals with mathematical foundations of signal processing, more precisely time-frequency analysis.

Given an analogue signal, we want to obtain local information about its frequency contents in a given, short time interval. This means, we either cut out a part of the signal or we damp it with a nice function, such as a Gauss function. Then, we want to decompose the signal into simple building blocks of (pure) sinusoids, i.e., simple waves with integer frequencies. This allows us to take local measurements of a signal and to extract the amplitudes of the active frequency bands. These measurements, called coefficients, can then be transmitted to a receiver where the local information is used to reconstruct the signal.

One way to mathematically implement this procedure are so-called Gabor systems, named after the physics Nobel laureate D. Gabor who already studied this theory in 1946. These and similar systems are now state-of-the-art in digital communication and are used e.g. in WLAN or 4G/5G technology (to name only a few).

Even though the theory is many decades old, there are still many open mathematical problems. One of the most important questions is how many samples per second one needs and how to divide the frequency bands in order to obtain a stable transmission process and a stable reconstruction. One of the very first results in this direction was the Whittaker-Nyquist-Kotelnikov-Shannon (WNKS) sampling theorem, which had a huge impact on the theory of digital communication and was already well-known in the 1930s. It gives a sufficient condition on how to digitize an analogue signal without losing any information, under the hypothesis that the signal is band-limited. This means that the amplitudes of sinusoids above a certain threshold level are zero. Gabor systems do not need this assumption and the WNKS sampling theorem can be obtained as a special case.

The project deals with timely questions of finding similar sampling theorems for certain Gabor systems, but it also deals with the question of finding “optimal” sampling strategies within a certain class of Gabor systems. The question on optimal sampling strategies as well as on sampling theorems does not always have a general answer and may depend on the specific situation and the chosen system.

So far, most of the general mathematical results are only established for 1-dimensional signals (which cover the measurement of sound waves, EEG, ECG, etc.) and it is not clear whether the mathematical theorems hold in full generality for 2-, or 3- (or higher-) dimensional signals. To advance the theory for higher-dimensional signals is another aim of this project.

Lastly, it has been observed that there are strong connections between Gabor systems and number theory. The study of some of these connections will be carried out in the project and is of independent, mathematical interest.